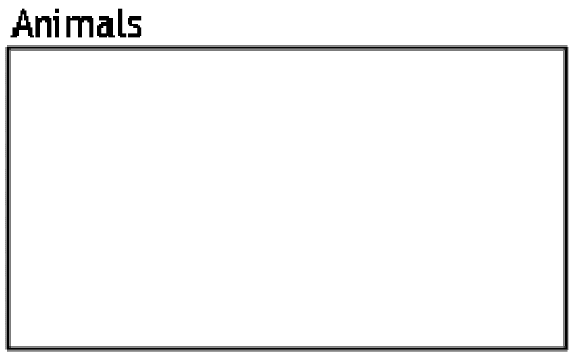
Venn Diagrams

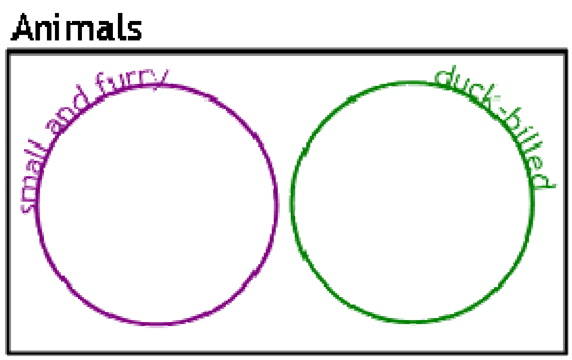
Venn diagrams were invented by a guy named John Venn as a way of picturing relationships between different groups of things. (Inventing this type of diagram was, apparently, pretty much all he ever accomplished. To add insult to injury, much of what we refer to as "Venn diagrams" are actually "Euler" diagrams. But we'll stick with the usual "Venn" terminology for the purposes of this lesson.) Since the mathematical term for "a group of things" is "a set", Venn diagrams can be used to illustrate both set relationships and logical relationships.

To draw a Venn diagram, you first draw a rectangle which is called your "universe". In the context of Venn diagrams, the universe is not "everything", but "everything you're dealing with right now". Let's deal with the following list of things: moles, swans, rabid skunks, geese, worms, horses, Edmontosorum (a variety of duck-billed dinosaurs), platypusses, and a very fat cat.

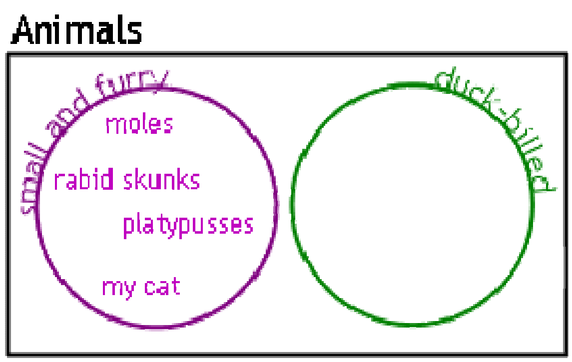
We'll call our universe "Animals":



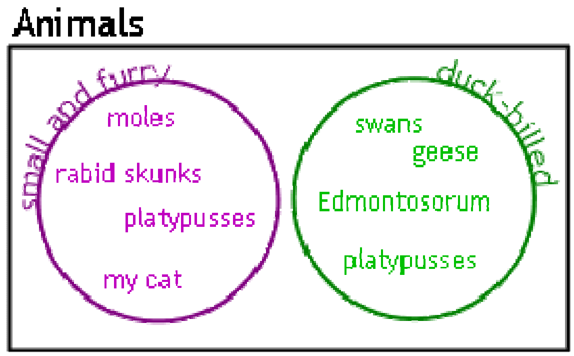
Let's say we want to classify things according to being small and furry or being a duck-bill. We draw circles to display our classifications:



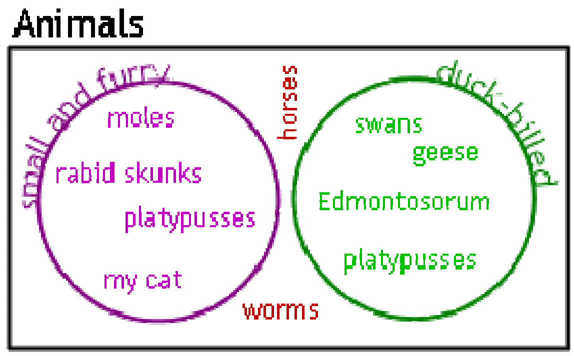
Now we'll fill in, or "populate", the diagram. Moles, rabid skunks, platypusses, and my (dear departed) cat are all small and furry:



Swans, geese, platypusses, and Edmontosorum are all duck-bills:

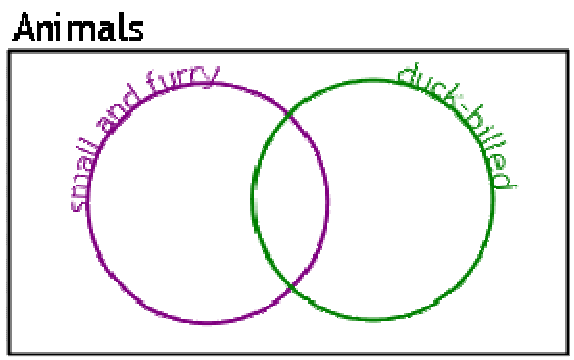


Worms are small but not furry and horses are furry but not small, and neither is a duck-bill. However, they are animals; they fit inside our universe, but outside the circles.

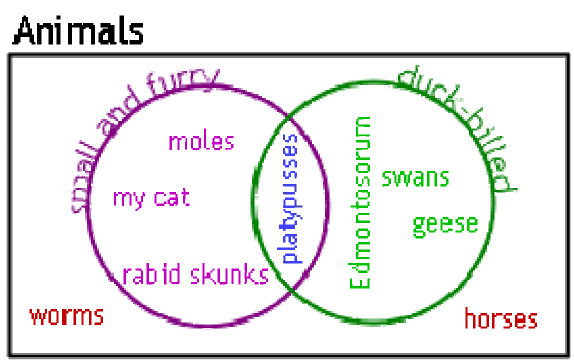


Notice that "platypusses" is listed in both of the circles. The point of Venn diagrams is that we can show this overlap in set membership by overlapping these circles.

In other words, we really should have drawn the circles overlapped, like this:



Now when we populate the Venn diagram, we'll only have to write "platypusses" once, in the overlap:



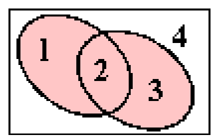
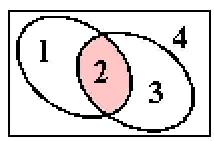
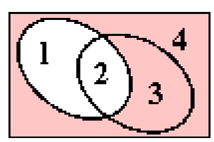
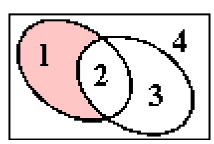
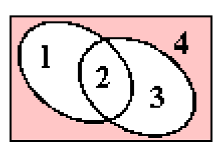
The overlap of the two circles, containing only platypusses, is called the "intersection" of the two sets.

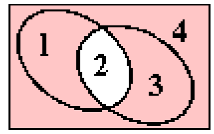
When drawing Venn diagrams, you will probably always be dealing with two or three overlapping circles, since having only one circle would be boring, and having four or more circles quickly becomes astonishingly complicated.

Set Notation and Venn Diagrams

The following examples should help you understand the notation, terminology, and concepts related to Venn diagrams and set notation.

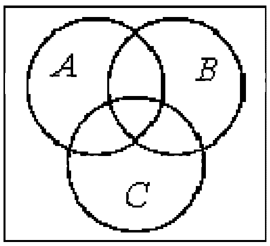
Let's say that our universe contains the numbers 1, 2, 3, and 4. Let A be the set containing the numbers 1 and 2; that is, $A = \{1, 2\}$. (Warning: The curly braces are the customary notation for sets. Do not use parentheses or square brackets.) Let B be the set containing the numbers 2 and 3; that is, $B = \{2, 3\}$. Then we have the following relationships, with pinkish shading marking the solution "regions" in the Venn diagrams:

set notation	pronunciation	meaning	Venn diagram	answer
$A \cup B$	"A union B"	everything that is in either of the sets		$\{1, 2, 3\}$
$A \wedge B$ or $A \cap B$	"A intersect B"	only the things that are in both of the sets		$\{2\}$
A^c or $\sim A$	"A complement", or "not A"	everything in the universe outside of A		$\{3, 4\}$
$A - B$	"A minus B", or "A complement B"	everything in A except for anything in its overlap with B		$\{1\}$
$\sim(A \cup B)$	"not (A union B)"	everything outside A and B		$\{4\}$
$\sim(A \wedge B)$	"not (A intersect B)"	everything outside of the overlap		$\{1, 3, 4\}$

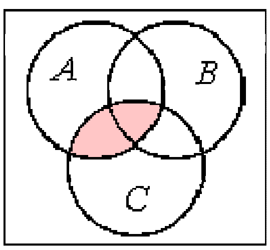
or ~(A ∩ B)		of A and B		

There are gazillions of other possibilities for set combinations and relationships, but these are among the simplest and most common. Note that different texts use different set notation, so you should not be at all surprised if your text uses still other symbols than those used above. But while the notation may differ, the concepts will be the same. By the way, as you probably noticed, your Venn-diagram "circles" don't have to be perfectly round; ellipses will do just fine.

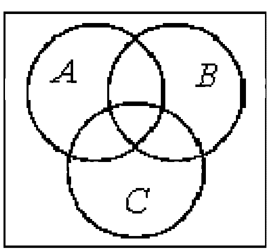
- **Given the following Venn diagram, shade in A ^ C.**



The intersection of A and C is just the overlap between those two circles, so:

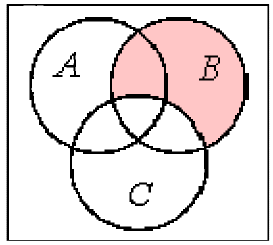


- **Given the following Venn diagram, shade in A U(B – C).**

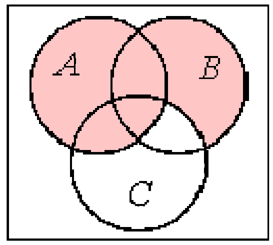


As usual when faced with [parentheses](), I'll work from the inside out.

I'll first find $B - C$.
"B complement$C$" means I take B and then throw out its overlap with C, which gives me this:

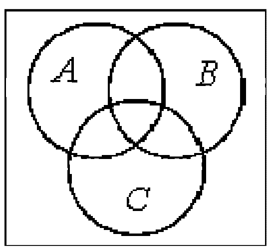


Now I have to union this with A:



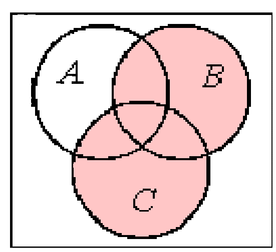
Note that unioning with A put some of C (that is, some of what I'd cut out when I did "$B - C$") back into the answer. This is okay. Just because we threw out C at one point, doesn't mean that it all has to stay out forever.

- **Given the following Venn diagram, shade in ~$[(B \mathrm{U} C) - A]$.**

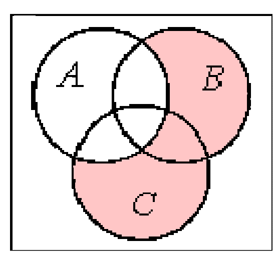


As usual when dealing with nested grouping symbols, I'll work from the inside out.

The union of B and C shades both circles fully:



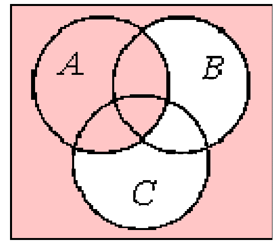
Now I'll do the "complement A" part by cutting out the overlap with A:



The tilde ("TILL-duh") is the wiggly "~" character at the beginning of ~$[(B \mathrm{U} C) - A]$; on

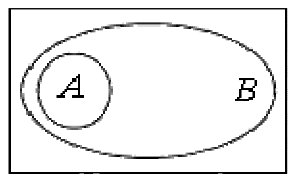
your keyboard, the tilde is probably located at or near the left-hand end of the row of numbers. The tilde, in this context, says that I now want to find the complement of what I've shaded. There are two kinds of complement in this problem. The set-subtraction complement in the previous step throws out any overlap between two given sets. But the kind of complement we see in this step, the "not" complement, means "throw out everything you have now and take everything else in the universe".

Practically speaking, the "not" complement with the tilde says to reverse the shading:



While Venn diagrams are commonly used for set intersections, unions, and complements, they can also be used to show subsets.

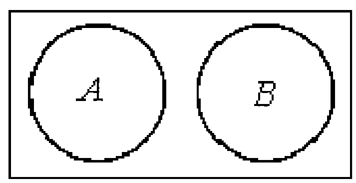
For instance, the picture to the right displays that A is a subset of B:



$A \subset B$

As you can see above, a subset is a set which is entirely contained within another set. For instance, every set in a Venn diagram is a subset of that diagram's universe.

Venn diagrams can also demonstrate "disjoint" sets. In the graphic to the right, A and B are disjoint:



$A \wedge B = \text{Ø}$

That is, disjoint sets have no overlap; their intersection is empty. There is a special notation for this "empty set", by the way: "Ø". (Unless you have an odd computer set-up, the preceding character looks like an "O" with a forward slash through it. If you're on a PC, you can type this "empty set" character by holding down the "ALT" key and typing "0216" on the numeric keypad.) This "Ø" character is pronounced as "the empty set".

An illustration of a use of these set relationships would be the manner in which some search engines process searches:

- If you type "cats AND dogs" into the search box, a search engine using this syntax (called "Boolean" logic) will return all web pages that contain both the word "cats" and the word "dogs". This corresponds to the set "C ^ D".

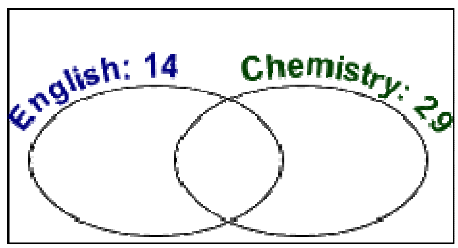
- If, on the other hand, you type "cats OR dogs", the search engine will return web pages that contain either the word "cats" or the word "dog" (or both, because the mathematical meaning of "or" is "inclusive"). This "or" statement corresponds to the set "C U D".
- If you type "cats NOT dogs", the search engine will return pages containing the word "cats", but only after discarding all the pages which also contain the word "dogs". This corresponds to the set "$C - D$".

Venn diagram word problems generally give you two or three classifications and a bunch of numbers. You then have to use the given information to populate the diagram and figure out the remaining information. For instance:

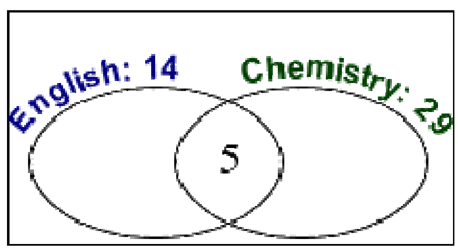
- **Out of forty students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes, how many students are in neither class? How many are in either class? What is the probability that a randomly-chosen student from this group is taking only the Chemistry class?**

  There are two classifications in this universe: English students and Chemistry students.

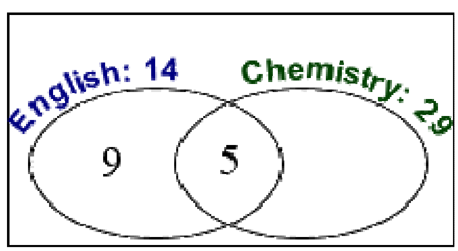
  First I'll draw my universe for the forty students, with two overlapping circles labelled with the total in each:



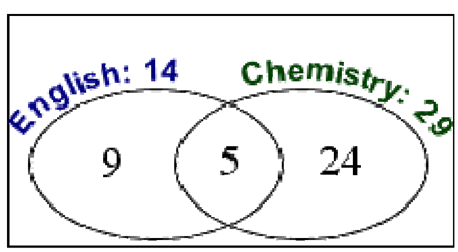
  Since five students are taking both classes, I'll put "5" in the overlap:



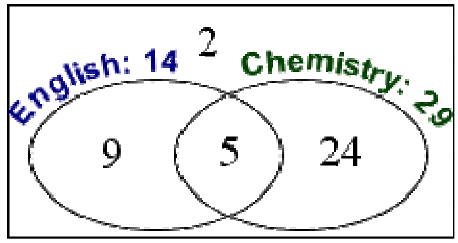
  I've now accounted for five of the 14English students, leaving nine students taking English but not Chemistry, so I'll put "9" in the "English only" part of the "English" circle:



  I've also accounted for five of the 29Chemistry students, leaving 24 students taking Chemistry but not English, so I'll put "24" in the "Chemistry only" part of the "Chemistry" circle:



This tells me that a total of $9 + 5 + 24 = 38$ students are in either English or Chemistry (or both). This leaves two students unaccounted for, so they must be the ones taking neither class.



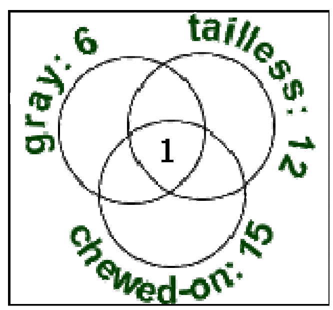
From this populated Venn diagram, I can get the answers to the questions.

Two students are taking neither class.
There are 38 students in at least one of the classes.
There is a $24/40 = 0.6 = 60\%$ probability that a randomly-chosen student in this group is taking Chemistry but not English.

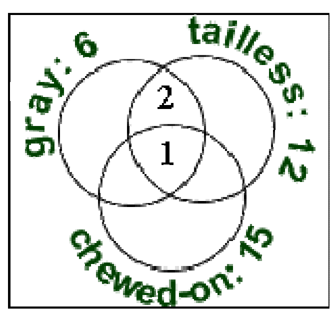
- **Suppose I discovered that my cat had a taste for the adorable little geckoes that live in the bushes and vines in my yard, back when I lived in Arizona. In one month, suppose he deposited the following on my carpet: six gray geckoes, twelve geckoes that had dropped their tails in an effort to escape capture, and fifteen geckoes that he'd chewed on a little. Only one of the geckoes was gray, chewed on, and tailless; two were gray and tailless but not chewed on; two were gray and chewed on but not tailless. If there were a total of 24 geckoes left on my carpet that month, and all of the geckoes were at least one of "gray", "tailless", and "chewed on", how many were tailless and chewed on but not gray?**

If I work through this step-by-step, using what I've been given, I can figure out what I need in order to answer the question. This is a problem that takes some time and a few steps to solve.

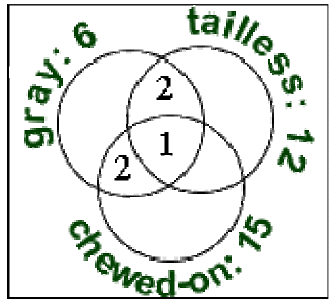
There was one gecko that was gray, tailless, and chewed on, so I'll draw my Venn diagram with three overlapping circles, and put "1" in the center overlap:



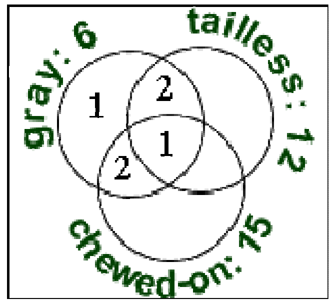
Two were gray and tailless but not chewed on, so "2" goes in the rest of the overlap between "gray" and "tailless".



Two were gray and chewed on but not tailless, so "2" goes in the rest of the overlap between "gray" and "chewed-on".

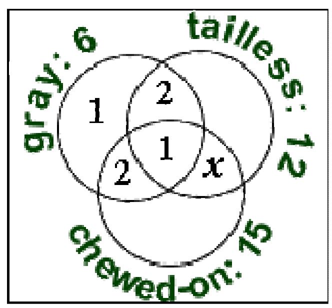


Since a total of six were gray, and since $2 + 1 + 2 = 5$ have already been accounted for, this tells me that there was only one left that was only gray.



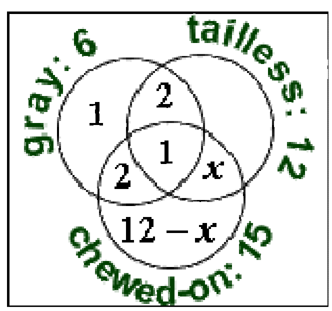
This leaves me needing to know how many were tailless and chewed on but not gray, which is what the problem asks for. Since I don't know how many were only chewed on or only tailless, I cannot yet figure out the answer.

I'll let "x" stand for this unknown number of tailless, chewed-on geckoes.

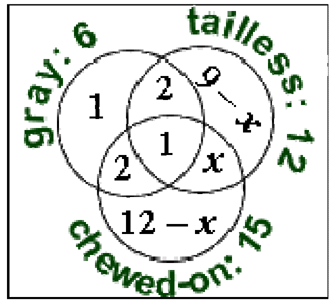


I do know the total number of chewed geckoes (15) and the total number of tailless geckoes (12). This gives me:

only chewed on: $15 - 2 - 1 - x = 12 - x$



only tailless: $12 - 2 - 1 - x = 9 - x$



There were a total of 24 geckoes for the month, so adding up all the sections of the diagram's circles gives me:

$$1 + 2 + 1 + 2 + x + (12 - x) + (9 - x) = 27 - x = 24$$

Solving, I get that $x = 3$.

Three geckoes were tailless and chewed on but not gray.